Formelsammlung HF II

S-Parameter ↔ Spg

$\frac{U_{hi}}{I_{hi}} = Z_L$ $\quad U_i = U_{hi} + U_{ri}$ $\quad U_{hi} = \frac{1}{2}(U_i + I_{hi} \cdot Z_L$

$\frac{U_{ri}}{I_{ri}} = Z_L$ $\quad I_i = I_{hi} - I_{ri}$ $\quad U_{ri} = \frac{1}{2}(U_i - I_i \cdot Z_L)$

$= \frac{1}{Z_L}(U_{hi} - U_{ri})$

Wellenmatrizen

Streumatrix

Def.: $\underline{U}_r = \underline{\underline{S}} \cdot \underline{U}_h$

$\Gamma_a = \frac{Z_a - Z_L}{Z_a + Z_L}$ … Reflexionsfaktor

Verlustlos: $\underline{\underline{S}} \cdot \underline{\underline{S}}^{*T} = \underline{\underline{I}}$

$\sum_{i=1}^{n} |S_{ij}|^2 = 1$

$\Gamma_1 = S_{11} + \frac{S_{12} S_{21} \Gamma_e}{1 - S_{22}\Gamma_e}$ $\qquad \Gamma_2 = S_{22} + \frac{S_{12} S_{21} \Gamma_e}{1 - S_{11}\Gamma_e}$

Bestimmung d. Streuparameter:

*) $S_{11} = \Gamma_1 = \frac{Z_1 - Z_L}{Z_1 + Z_L}$

*) $S_{21} = \frac{U_{r2}}{U_{h1}}\Big|_{U_{h2}=0} = \frac{U_2}{U_1}(1 + \overset{S_{11}}{\Gamma_1})$

Spg-Übertragung: $\frac{U_2}{U_1} = \frac{S_{21}(1 + \Gamma_e)}{(1 - S_{22}\Gamma_e)(1 + \Gamma_1)}$ mit $\Gamma_1 = S_{11} + \frac{S_{12} S_{21} \Gamma_e}{1 - S_{22}\Gamma_e}$

WW-Anpassung

$U_{h1} = \frac{U_0}{2}$ $\qquad \Gamma_1 = S_{11}$

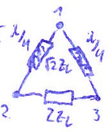
$\frac{U_2}{U_1} = \frac{S_{21}}{1 + S_{11}}$ $\qquad \Gamma_2 = S_{22}$

$g_T = |S_{21}|^2$

Betriebsleistungs-Verstärkung: $g_T = \frac{P_2}{P_{1,max}} = \frac{|S_{21}|^2 (1 - |\Gamma_i|^2)(1 - |\Gamma_e|)^2}{|(1 - S_{11}\Gamma_i)(1 - S_{22}\Gamma_e) - S_{12} S_{21} \Gamma_e \Gamma_i|^2} = |A_B|^2$

Ltg-Schaltungen

Wilkinsonteiler



$\underline{\underline{S}} = -\frac{j}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

90°-Hybrid-Koppler

$\underline{\underline{S}} = -\frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1 & j \\ 0 & 0 & j & 1 \\ 1 & j & 0 & 0 \\ j & 1 & 0 & 0 \end{bmatrix}$

Magic Tee $\underline{\underline{S}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$

Richtkoppler $\quad \underline{\underline{S}} = \begin{bmatrix} 0 & k & -j\sqrt{1-k^2} & 0 \\ k & 0 & 0 & -j\sqrt{\ } \\ -j\sqrt{\ } & 0 & 0 & k \\ 0 & -j\sqrt{\ } & k & 0 \end{bmatrix}$

3dB: $\underline{\underline{S}}_{3dB} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & -j & 0 \\ 1 & 0 & 0 & -j \\ -j & 0 & 0 & 1 \\ 0 & -j & 1 & 0 \end{bmatrix}$

Zudem gilt: $S_{21} = \frac{jk \sin(\beta l)}{\sqrt{1-k^2}\cos(\beta l) + j\sin(\beta l)}$

$S_{31} = \frac{\sqrt{1-k^2}}{\sqrt{1-k^2}\cos(\beta l) + j\sin(\beta l)}$

Gekoppelte Ltg.: $k = \frac{M'}{L'} = \frac{C_{12}'}{C' + C_{12}'} = \frac{Z_{Le} - Z_{Lo}}{Z_{Le} + Z_{Lo}}$

$Z_L = \sqrt{Z_{Le} Z_{Lo}}$ $\quad Z_{Le} = Z_L \sqrt{\frac{1+k}{1-k}}$ $\quad Z_{Lo} = Z_L \sqrt{\frac{1-k}{1+k}}$

Dämpfungen:

Koppeldämpfung: $a_k = -10\log\frac{P_2}{P_{1,max}} = -20\log|S_{21}|$

Durchgangs~ : $a_B = -10\log\frac{P_3}{P_{1,max}} = -20\log|S_{31}|$

Richt~ : $a_R = -10\log\frac{P_4}{P_2} = -20\log\frac{|S_{41}|}{|S_{21}|}$

Transmissionsmatrix

$\begin{pmatrix} U_{r1} \\ U_{h1} \end{pmatrix} = \underline{\underline{T}} \cdot \begin{pmatrix} U_{h2} \\ U_{r2} \end{pmatrix}$ bei Kettenschaltungen: $\underline{\underline{T}}_g = \prod_{i=1}^{n} \underline{\underline{T}}_i$

Umrechnungen:

S ↔ Y:

$S_{11} = [(1 - y_{11})(1 + y_{22}) + y_{12} y_{21}] / N_1$

$S_{12} = -2 y_{12} / N_1$

$S_{21} = -2 y_{21} / N_1$

$S_{22} = [(1 + y_{11})(1 - y_{22}) + y_{12} y_{21}] / N_1$

$N_1 = (1 + y_{11})(1 + y_{22}) - y_{12} y_{21}$

$y_{11} = [(1 - S_{11})(1 + S_{22}) + S_{12} S_{21}] / N_2$

$y_{12} = -2 S_{12} / N_2$

$y_{21} = -2 S_{21} / N_2$

$y_{22} = [(1 + S_{11})(1 - S_{22}) + S_{12} S_{21}] / N_2$

$N_2 = (1 + S_{11})(1 + S_{22}) - S_{12} S_{21}$

S ↔ T:

$S_{11} = T_{12} / T_{22}$

$S_{12} = T_{11} - T_{12} \cdot \frac{T_{21}}{T_{22}} = \det \underline{\underline{T}} / T_{22}$

$S_{21} = 1 / T_{22}$

$S_{22} = -T_{21} / T_{22}$

$T_{11} = S_{12} - S_{11} S_{22} / S_{21} = -\det \underline{\underline{S}} / S_{21}$

$T_{12} = S_{11} / S_{21}$

$T_{21} = -S_{22} / S_{21}$

$T_{22} = 1 / S_{21}$

Wellen

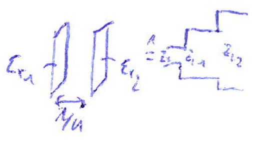
$Z_{F0} = 120\pi$

$Z_{FH} = Z_{F0} \cdot \frac{\lambda_H}{\lambda_0}$ $\quad v_p = f \cdot \lambda_H = c_0 \frac{\lambda_H}{\lambda_0}$

$\lambda_H = \frac{\lambda_0}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}}$ $\quad v_E = \frac{c_0^2}{v_p}$

Senkrechter Einfall $\quad \Gamma(0) = \frac{\sqrt{\frac{\mu_r}{\varepsilon_r}} - 1}{\sqrt{\frac{\mu_r}{\varepsilon_r}} + 1}$

$\lambda/4$-Absorber:



$Z = \frac{Z_{L1}^2}{Z_{L2}} \overset{!}{=} Z_L$

$\to Z_{L1} = \sqrt{Z_L Z_{L2}}$

$\varepsilon_{r1} = \sqrt{\varepsilon_{r2}}$

Leitende Ebene:

Stromdichte $\vec{J}_A = \vec{n} \times \vec{H}$

$\Gamma(0) = -1$

$U_r = -U_h$

$E_r = -E_h$

$H_{-x}(0) = 2 \cdot \frac{E(0)}{Z_{F0}}$

E-Wellen

E_{mn}: $H_z = 0$

$E_z = E_0 \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} e^{-j\beta z}$

$E_x = -j\frac{\beta}{\beta_c^2}\frac{m\pi}{a} E_0 \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} e^{-j\beta z}$

$E_y = -j\frac{\beta}{\beta_c^2}\frac{n\pi}{b} E_0 \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} e^{-j\beta z}$

$H_x = -\frac{E_y}{Z_{FE}}$ $\quad H_y = \frac{E_x}{Z_{FE}}$

$\beta_c = \frac{2\pi}{\lambda_c}$

$\lambda_c = \frac{1}{\sqrt{\left(\frac{m}{2a}\right)^2 + \left(\frac{n}{2b}\right)^2}}$

H-Wellen

H_{mn}

$$E_z = 0$$

$$H_z = H_0 \cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b} e^{-j\beta z}$$

$$H_y = j\frac{\beta}{\beta_c^2}\frac{n\pi}{b} H_0 \cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b} e^{-j\beta z}$$

$$H_x = j\frac{\beta}{\beta_c^2}\frac{m\pi}{a} H_0 \sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b} e^{-j\beta z}$$

$$E_y = -H_x \cdot Z_{FH}$$

$$E_x = H_y \cdot Z_{FH}$$

$$\lambda_c = \frac{1}{\sqrt{\left(\frac{m}{2a}\right)^2 + \left(\frac{n}{2b}\right)^2}}$$

H_{10}

$$\lambda_c = 2a \rightarrow f_c = \frac{c_0}{2a}$$

Eindeutig für $a \leq \lambda_0 \leq 2a$

Leistung: $P = \frac{ab}{4}\frac{|E_0|^2}{Z_{FH}}$

$$S_z = \frac{1}{2}\frac{|E_0|^2}{Z_{FH}}\left(\sin\frac{\pi x}{a}\right)^2$$

Verluste

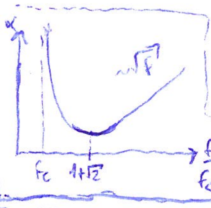
$1\,Np \hat{=} 8{,}686\,dB$

Wandverluste: $\alpha = \frac{R_\square}{Z_{F_0}} \frac{\frac{1}{b}+\frac{2}{a}\left(\frac{\lambda_0}{2a}\right)^2}{\sqrt{1-\left(\frac{\lambda_0}{a}\right)^2}} \left[\frac{Np}{m}\right]$

$$= \frac{1+\left(\frac{\lambda_0}{a}\right)^2}{\sqrt{a^2+\lambda_0^2}} \text{ für } a=2b$$

$f < f_c$: $\alpha = \frac{2\pi}{\lambda_c}\sqrt{1-\left(\frac{\lambda_c}{\lambda_0}\right)^2} \xrightarrow{f\to 0} \frac{2\pi}{\lambda_c}$

$\hookrightarrow E = E_0 e^{-\alpha \Delta z}$



$\rightarrow \frac{f}{f_c} = \frac{2a}{\lambda_0}$ (für $a = 2b$)

H-Felder: $Z_{FH} = jZ_{F_0}\frac{1}{\sqrt{\left(\frac{\lambda_0}{\lambda_c}\right)^2-1}}$ → 90° Phase → kein E-Transport

$$R_\square = \sqrt{\frac{\omega\mu}{2\kappa}}$$

$$\frac{P_1}{P_2} = \sqrt{\frac{\kappa_2}{\kappa_1}}$$

Mikrowellenresonatoren

H_{mnp}, E_{mnp}

$$\lambda_R = \frac{\lambda_c}{\sqrt{1+\left(p\frac{\lambda_c}{2c}\right)^2}}$$

$$\left[\text{bei Rechteck: } \lambda_R = \frac{1}{\sqrt{\left(\frac{m}{2a}\right)^2+\left(\frac{n}{2b}\right)^2+\left(\frac{p}{2c}\right)^2}}\right]$$

Gesp. Energie

$$W_{max,el} = \int_V \frac{1}{2}\varepsilon_0 |e_{max}|^2 dV$$

$$W_{max,mag} = \int_V \frac{1}{2}\mu_0 |h_{max}|^2 dV$$

(über ein Halbwelle, z.B. $0 \leq z \leq p\frac{\lambda_H}{2}$)

$\left[H_{101}: W_{el,max} = \frac{1}{2}\varepsilon_0 |E_h|^2 abc\right.$

Unbelastete Güte

$$Q_U = \frac{\omega_R W_{max}}{P_V}$$

Quader (H_{101}): $Q_U = \frac{\lambda_R}{\delta}\frac{b(a^2+c^2)^{3/2}}{2\left[c^3(a+2b)+a^3(c+2b)\right]} \left[\underset{b=a/2}{\overset{a=c}{=}} \frac{\sqrt{2}}{8}\right]$

$$\delta = \sqrt{\frac{2}{\omega\mu\kappa}}$$

(Skin-Effekt)

Zylinder: E_{101}: $Q_U = 0{,}38\frac{\lambda_R}{\delta}\frac{1}{1+\frac{1}{2}\frac{D}{c}}$

H_{011}: $Q_U = 0{,}61\frac{\lambda_R}{\delta}\frac{\left[1+0{,}17\left(\frac{D}{c}\right)^2\right]^{3/2}}{1+0{,}17\left(\frac{D}{c}\right)^3}$

Induktive Ankopplung

$$A_K = \frac{\lambda_H}{4\pi}\sqrt{\frac{R_p^* W_R \varepsilon_0 abc}{Q_U}}$$

Dielektrische Resonatoren

$$r = \frac{\sqrt{\varepsilon_r}-1}{\sqrt{\varepsilon_r}+1} \xrightarrow{\varepsilon_r\to\infty} 1$$

ESB: $Z_0/\sqrt{\varepsilon_r}$, Z_0

→ magnet. Wand

→ $H_t = 0, E_n = 0$ (duale Felder) $E_1 \circ\!\!-\!\!\circ H_2$, $E_2 \circ\!\!-\!\!\circ H_1$

$H_{01\delta}$: $\lambda_R(H_{01\delta}) = \lambda_R(H_{020}) \cdot \Psi(\varepsilon_r, \frac{D}{c})$

mit $\Psi(\varepsilon_r, D/c) = \frac{1}{1{,}02+0{,}15\,D/c}$

$$Q_U \lessapprox \frac{1}{\tan\delta_\varepsilon}$$

Modenfilter

E-Mode: Längsströme b

H_{20}: Querströme a

H_{01}: Längsströme b

Leitungstheorie

$E_y \circ\!\!-\!\!\circ U$

$H_x \circ\!\!-\!\!\circ I$

$$I_h = \frac{E_h}{Z_{FH}}\frac{2a}{\pi}$$

$$Z_L = \frac{\pi^2}{8}\frac{b}{a}Z_{FH}$$

~~Querschnittssprung~~: